Fourier Neural Operators for Floquet Quantum Dynamics

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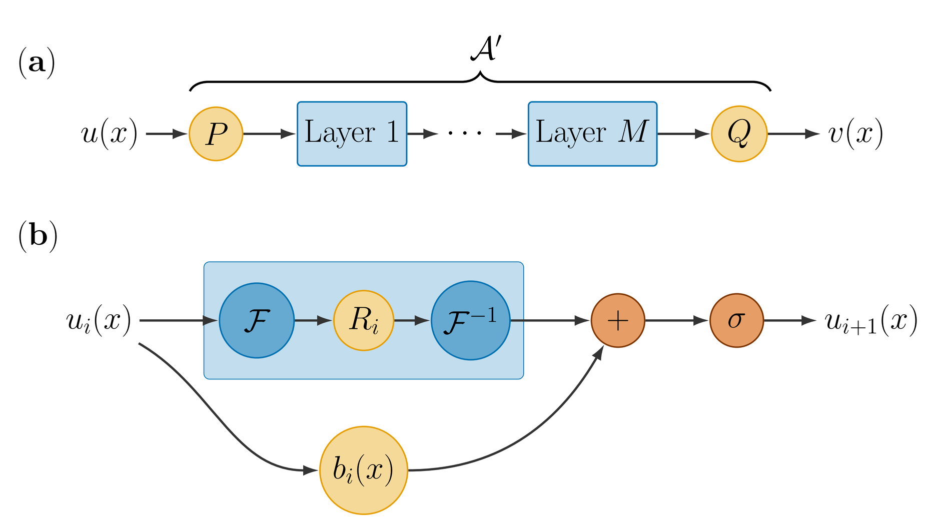
Motivation

- Floquet (time-periodic) system is a versatile platform for realizing exotic, non-equilibrium physics.
- Numerical simulation remains challenging, particularly due to growth of Hilbert space dimension and entanglement.
- Experimental advance makes it possible to access measurements on larger system sizes, but for times limited by the coherence window.
- Can we use a data-driven approach to infer long-time Floquet dynamics from short-time, local measurements?

Fourier Neural Operators (FNO)



- Neural operators learn mappings **between functions**, through a sequence of layers [(a)].
- Each layer of FNO performs a **spectral convolution** and linear transformation on the input function [(b)]:

$$u_{i+1}(x) = \sigma\left(\mathcal{F}^{-1}(R_i \cdot \mathcal{F}(u_i))(x) + b_i(x) \cdot u_i(x)\right)$$

- FNO is **discretization invariant:** it can be trained on a coarser grid and be evaluated on a finer one.

Data required: **Hamiltonian parameters** $H(t)$; **effective Floquet Hamiltonian** $H_F(t)$; expectation values of **one- and two-local observables** $\{\langle B_i(t)\rangle\}$.

Summary of Learning tasks:

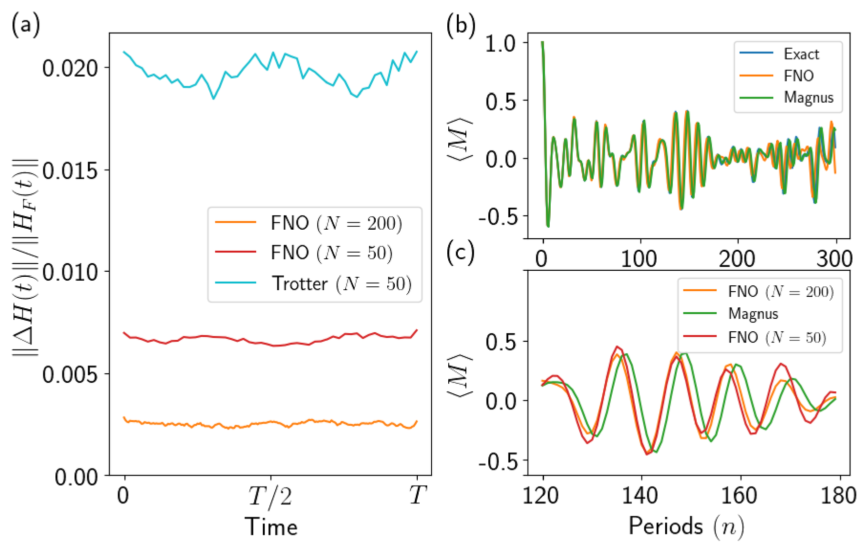
	Input Function	Output Function	Function Domain	Training Data
Paradigm 1	$H(t)$	$H_F(t)$	$(0, T)$	$H(t), H_F(t)$
Paradigm 2	$H(t)$	$\langle B_i(t)\rangle$	$(0, nT)$	$H(t), \langle B_i(t)\rangle$
	$\langle B_{\text{in}}(t)\rangle$	$\langle B_{\text{out}}(t)\rangle$	$(0, nT)$	$\langle B_i(t)\rangle$
Paradigm 3	$H(t)$	$c(t)$	$(0, T)$	$H(t), \langle B_i(t)\rangle, \text{Tr}(\rho B_i)$

Paradigm 1: Learning Floquet Hamiltonian

Model Hamiltonian (TFIM with integrability-breaking, **spatio-temporal** disorder):

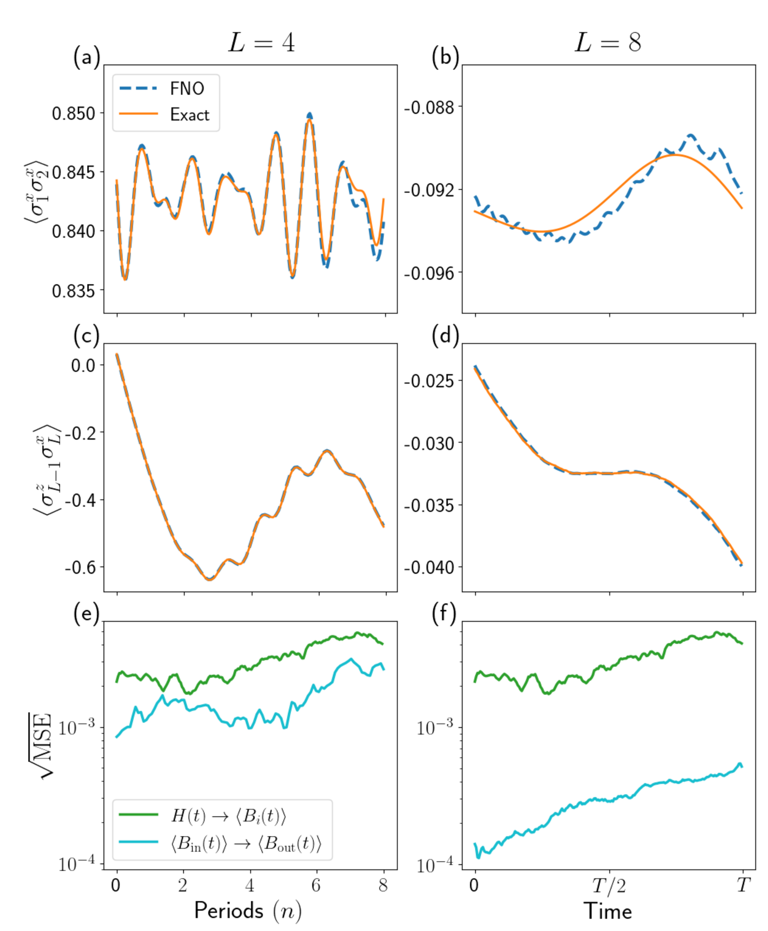
$$H(t) = \sum_{i=1}^{L} J_i X_i X_{i+1} + A_i \cos(\omega t) Z_i + h_x(i,t) X_i.$$

Use FNO to learn the mapping from Hamiltonian parameters to Floquet Hamiltonians, $H(t) \to H_F(t), t \in (0, 2\pi/\omega)$. Here $H_F(t) := \frac{i}{T}\log\left(\hat{\mathcal{T}} e^{-i\int_t^{t+T} H(t')dt'}\right)$.



FNO achieves **less error than equal-mesh Trotterization**, captures local observables **as accurately as Magnus expansion**, and is able to transfer learning to **a mesh four times denser.**

Paradigm 2: Learning Local Observables



FNO predicts expectation values of local observables from *either* **Hamiltonian parameters or partial measurements** (e.g. learning two-local from one-local measurements).

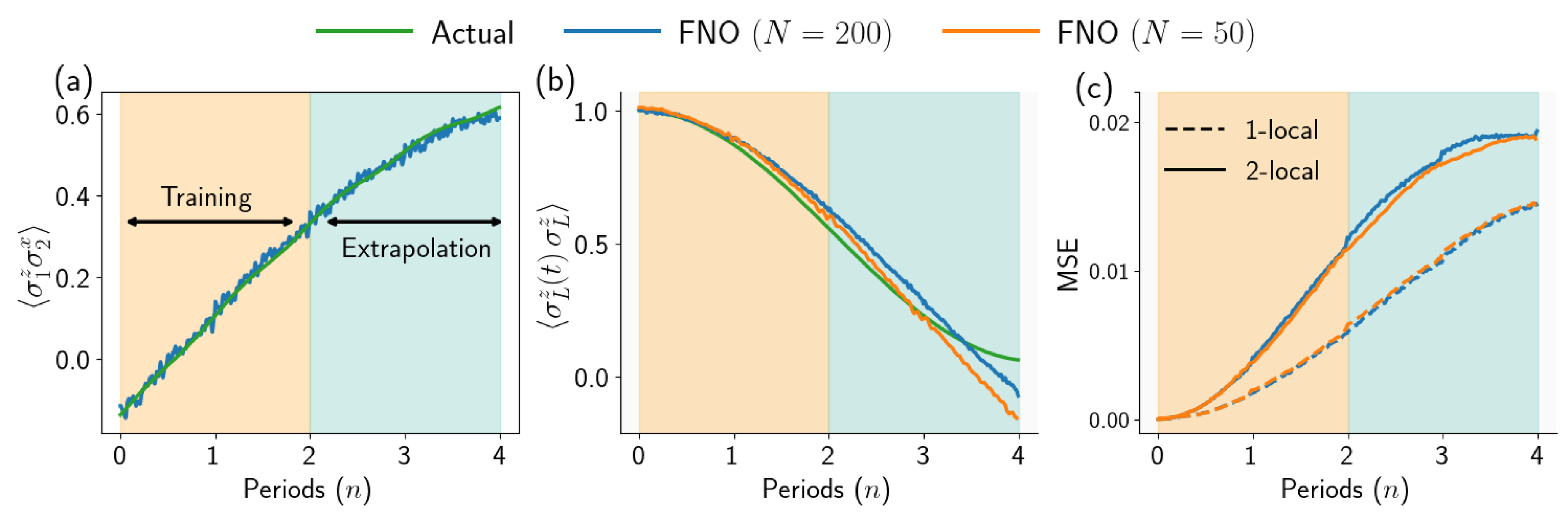
Consistently achieves **sub-percent relative error**, both for one period and over multiple cycles.

Paradigm 3: Capturing Operator Growth

FNO captures how local operators grow under Floquet dynamics.

Define $c_{ij}(t) = \frac{1}{2^L}\text{Tr}\left(B_j B_i(t)\right)$ as the matrix that encodes operator spreading. FNO learns the mapping from $H(t) \to c(t)$.

For **unseen Hamiltonians**, FNO can predict **local expectation values, autocorrelation functions, and time-evolved operators.**



All data required are **experimentally accessible**. Allows for **extrapolating** observables **beyond** the training window.

Summary and Outlook

- Fourier Neural Operator (FNO) is a powerful and versatile deep-learning computational surrogate for modeling time-periodic quantum dynamics.
- We demonstrate its applicability through three learning paradigms:
  - Predicting **effective Floquet Hamiltonians**, enabling predictions of stroboscopic measurements;
  - Predicting **expectation values of local observables**, from Hamiltonian parameters or partial measurements; and
  - Learning **operator growth dynamics** under time-periodic driving.
- FNO's complexity is expected to scale **polynomially** with system size; its training only requires **realistic, experimentally accessible** data.
- FNO could serve as a powerful surrogate for analyzing measurement data from NISQ devices, e.g. extrapolating beyond their coherence window.

Reference

Z. Qi, Y. Peng, C. Earls. Fourier Neural Operators for Time-Periodic Quantum Systems: Learning Floquet Hamiltonians, Observable Dynamics, and Operator Growth. arXiv preprint:2509.07084.