Real-Space Topological Invariant for Time-Quasiperiodic Majoranas

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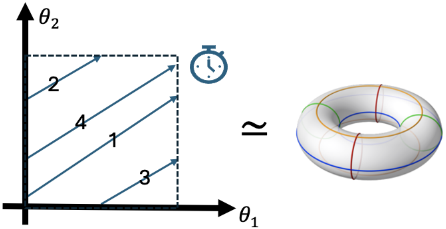
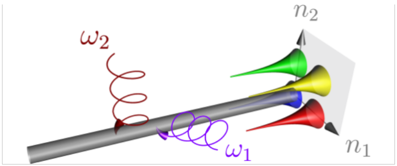
Time-Quasiperiodic Systems



- Generalization of Floquet (time-periodic) systems.
- e.g. $H(t) = f(\omega_1 t, \omega_2 t)$. ω_1 and ω_2 mutually irrational.
- Quasi-energy is defined modulo $n_1\omega_1 + n_2\omega_2$, $n_1, n_2 \in \mathbb{Z}$.
- For sufficiently large $|n_1|, |n_2|$, $n_1\omega_1 + n_2\omega_2$ can approach any real number $\rightarrow$ *dense* quasienergy spectrum.

Time-Quasiperiodic Majoranas



- Four particle-hole symmetric quasi-energies[1]:

$$\bar{\epsilon} = 0, \omega_1/2, \omega_2/2, (\omega_1 + \omega_2)/2.$$

- Question: How do we define a topological invariant in the gapless system?

[1] Y. Peng and G. Refael, PRB 98, 220509 (R), 2018.

Spectral Localizer

- Detects whether incompatible observables have approximate common eigenstates.
- For Hamiltonian H, position operator X, define localizer as:

$$L_{x,E}(X, H) = \kappa(X - xI) \otimes \Gamma_1 + (H - EI) \otimes \Gamma_2.$$

κ is a hyperparameter. $\Gamma^\dagger_{1,2} = \Gamma_{1,2}$, $\Gamma_1\Gamma_2 = -\Gamma_2\Gamma_1$, $\Gamma^2_{1,2} = I$.

- If there is a common eigenstate $|\psi\rangle$ of H and X:

$$H|\psi\rangle = E^*|\psi\rangle, \; X|\psi\rangle = x^*|\psi\rangle,$$

then L_{x^*,E^*} has a zero eigenvalue.

Topological Invariant

- Choose $\Gamma_1 = \sigma_x$, $\Gamma_2 = \sigma_y$.

$$L_{x,\epsilon}(X,K) = \begin{pmatrix} 0 & \kappa(X - xI) - i(K - \epsilon I) \\ \kappa(X - xI) + i(K - \epsilon I) & 0 \end{pmatrix}$$

- K is the enlarged, frequency-domain Hamiltonian.
- Symmetries of $L_{x,\epsilon}$:
  - Particle-hole: $(V_c \otimes \sigma_z)\, L^*_{x,\bar{\epsilon}}\, (V_c \otimes \sigma_z)^{-1} = -L_{x,\bar{\epsilon}}$.
  - Chiral: $(I \otimes \sigma_z)\, L_{x,\epsilon}\, (I \otimes \sigma_z)^{-1} = -L_{x,\epsilon}$.
- $L_{x,\epsilon}$ is in 0D Class BDI. Classified by a $\mathbb{Z}_2$ invariant:

$$C_{x,\bar{\epsilon}} = \text{sign}\left(\det\left(\kappa(X - xI) + i(K - \bar{\epsilon} I)\right)\right).$$

Interpretation using non-Hermitian Physics

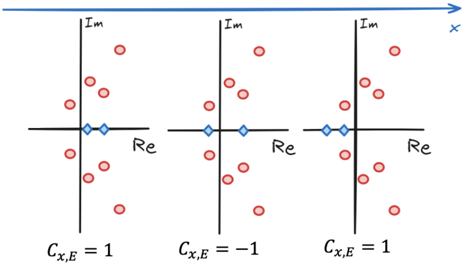
- Define $M_x(\kappa) := \kappa(X - xI) + i(K - \bar{\epsilon}I)$.
- $M_x(\kappa)$ is PT symmetric but non-Hermitian. It has two types of eigenvalues:
  - PT-breaking (bulk) states: $\Gamma_n \pm i\epsilon_n$.
  - PT-preserving (Majorana) states: (δ_L, δ_R).
- $C_{x,\epsilon} = \text{sign}(\det M) = \text{sign}(\prod_n (\Gamma_n \pm i\epsilon_n)\delta_L\delta_r) = \text{sign}(\delta_L\delta_R)$.

Intepretation, cont'd

- For small κ, first order perturbation gives:

$$\delta_{L,R} \simeq \kappa(\langle\psi_{L,R}| X |\psi_{L,R}\rangle - x)$$

- For x near either boundary, $\delta_{L,R}$ have the same sign, $C = 1$.
- For x near chain center, $\delta_L < 0, \delta_R > 0$, $C = -1$.
- Real-space signature of Majoranas: change of invariant from 1 to -1 as x goes from boundary into the bulk.



Hyperparameter κ

- How to choose the hyperparameter κ?
- Previous understanding: κ cannot be too large, often chosen empirically.[2]
- Connection with non-Hermitian physics: at $\kappa = \kappa_c$, $M(\kappa_c)$ is non-diagonalizable (an exceptional point).
- For $\kappa > \kappa_c$, $M(\kappa)$ becomes topologically distinct from $M(\kappa = 0)$. No PT-preserving modes (Majoranas) anymore.
- The criteria for choosing κ: no EP should be met as κ increases from zero. i.e. $\kappa < \kappa_c$.

[2] K. Dixon, T. Loring, A. Cerjan, PRL 131, 213801 (2023).

Numerical Example: Driven Kitaev Chain

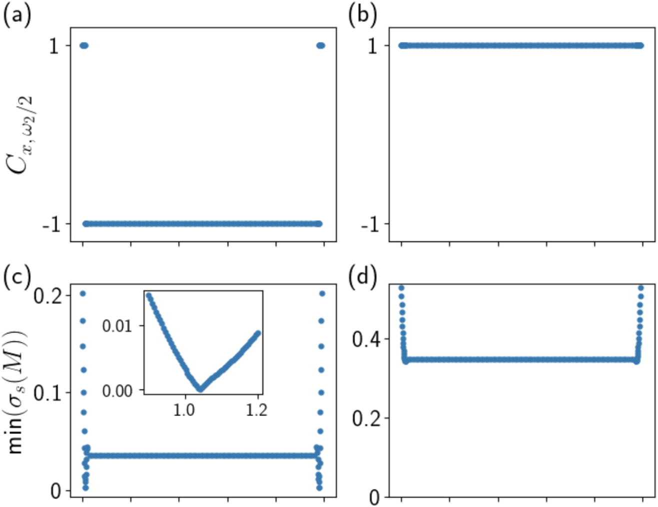
$$H(t) = H_K + V(\omega_1 t) + V(\omega_2 t)$$

$$H_K = -\mu \sum_{j=1}^{N} c_j^\dagger c_j - \sum_{j=1}^{N-1} [(J c_j^\dagger c_{j+1} + i\Delta c_j c_{j+1}) + \text{h.c.}]$$

$$V(\omega_i t; \Delta_i) = -i\Delta_i \sum_{j=1}^{N-1} (e^{-i\omega t} c_j c_{j+1} - e^{i\omega t} c_{j+1}^\dagger c_j^\dagger)$$

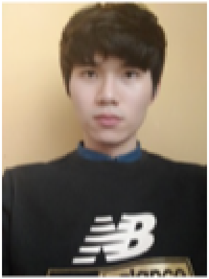
With Majorana

No Majorana

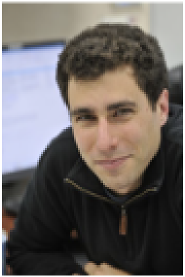


Collaborators

Yang Peng (CSUN)



Iryan Na (UCB)



Gil Refael (Caltech)

Summary & Outlook

- We defined a real-space topological invariant to detect time-quasiperiodic Majoranas using spectral localizer.
- Outlined a criteria for selecting the hyperparameter κ.
- Numerical study of driven Kitaev chain agrees with expectations.
- Physical setup for the non-Hermitian probe?

Phase Diagram